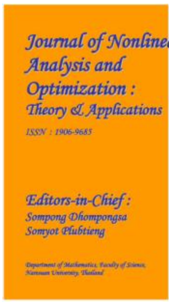
**J**ournal of **N**onlinear **A**nalysis and **O**ptimization
Vol. 14, Issue. 2, No. 4: 2023
ISSN : **1906-9685**




# PARTIAL DIFFERENTIAL EQUATION AND APPLICATION TO DECOMPOSITION OF ORGANIC MATTER IN 3D SOIL STRUCTURE

**S.Jeevitha.,** M.Sc.,M.phil.,B.Ed Assistant Professor Department of Mathematics Marudhar Kesari Jain College for Women, Vaniyambadi Email: jeevikiru@gmail.com Mobile No : 8870900142

## ABSTRACT

The aim of this paper is to present the over view of Partial Differential Equations and also discuss how concept of Partial Differential Equations is used in decomposition of organic matter in 3d soil structure.

**KEYWORDS** Partial Differential Equations , Modeling, 3D structures.

## INTRODUCTION

In mathematics a differential equation is an equation that relates one or more functions and their derivatives. In applications the functions generally represent physical quantities, the derivatives represent their rates of change, and the differential equation defines a relationship between are the two such relations common; therefore, differential equation play a prominent role in many disciplines. Including Engineering, Physics, Economics and Biology. Differential equations are of two types are Ordinary Differential Equations and Partial Differential Equations.

A Partial differential equation is a mathematical equation that involves two or more independent variables an unknown functions (dependent on their variables) and partial derivatives of the unknown function with respect to the independent variables.

## PARTIAL DIFFERENTIAL EQUATIONS

In Mathematics, a partial differential equation is one of the types of differential equations, in which the equation contains unknown multi variables with their partial derivatives. It is a special case of an ordinary differential equation. We are going to discuss what is a partial differential equation, how to represent it, its classification and type with more examples and solved problems.

A partial Differential Equation commonly denoted as PDE is a differential equation containing partial derivatives the dependent variable with more than one independent variable. A PDE for a function $u(x_1, x_2, \ldots\ldots, x_n)$ is an equation of the form

$$f\left(x_1, \ldots \ldots x_n; u, \left(\frac{\partial u}{\partial x}\right), \ldots., \left(\frac{\partial u}{\partial x_n}\right); \left(\frac{\partial^2 u}{\partial x_1 \partial x_1}\right), \ldots \ldots.., \frac{\partial^2 u}{\partial x_1 \partial x_n}, \ldots\right) = 0$$

The PDE is said to be linear if f is a linear function of u and its derivatives .The simple PDE is given by:

$$\frac{\partial u}{\partial x}(x, y) = 0$$

The above relation implies that the function u(x, y) is independent of X which is the reduced form of partial differential equation formula stated above. The order of PDE is the of the highest derivative term of the equation.

## REPRESENTATION OF PARTIAL DIFFERENTIAL EQUATIONS:

In PDEs we denote the partial derivatives using subscripts, such as :

$$u_x = \frac{\partial u}{\partial x}$$

$$u_{xx} = \frac{\partial^2 u}{\partial x^2}$$

$$u_{xy} = \frac{\partial^2 u}{\partial y\,\partial x} = \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial x}\right)$$

In some cases, like in physics when we learn about wave equations or sound equation, partial derivative $\partial$, is also represented by $\nabla$. (dell or nabla).

**CLASSIFICATION OF PARTIAL DIFFERENTIAL EQUATIONS**

Each type of PDE has certain functionalities that help to determine whether a particular finite element approach is appropriate to the problem being described by the PDE. The solution depends on the equation and several variables contain partial derivatives with respect to the variables. There are three types of second order PDEs in mechanics they are

- Elliptic PDE
- Parabolic PDE
- Hyperbolic PDE

Consider the example,

$$au_{xx} + bu_{yy} + cu_{yy} = 0\,, u = u(x, y)$$

For a given point (x, y), the equation is said to be elliptic if $b^2$-ac<0 which are used to describe the equations of elasticity without inertial terms. Hyperbolic PDEs describe the phenomena of wave propagation if it satisfies the condition $b^2 - ac = 0$. The heat conduction equation is an example of a parabolic PDE.

**TYPES OF PARTIAL DIFFERENTIAL EQUATION**

The different types of partial differential equation are,

- First-order partial differential equation
- Linear partial differential equation
- Quasi-linear Partial differential equation
- Homogenous partial differential equation

**TYPES OF PDES**

**FIRST-ORDER PARTIAL DIFFERENTIAL EQUATION**

In maths, when we speak about the first-order partial differential equation, then the equation has only the first derivative of the unknown function having 'm' variables. It is expressed in the form of

$$F(X_1, \dots\dots, X_m, u, u_{X1}, \dots\dots.., u_{Xm}) = 0$$

**LINEAR PARTIAL DIFFERENTIAL EQUATION**

If the dependent variable and all its partial derivatives occur linearly in any PDE then such an equation is called linear PDE otherwise a nonlinear PDE.

**QUASI-LINEAR PARTIAL DIFFERENTIAL EQUATION**

A PDE is said to be quasi-linear if all the terms with the highest order derivative all of dependent variable occur linearly, that is the coefficient of those terms are functions of only lower - order derivatives of the dependent variables.

**HOMOGENEOUS PARTIAL DIFFERENTIAL EQUATION**

If all the terms of a PDE contain the dependent variable or its partial derivatives. Then such a PDE IS called non-homogeneous partial differential equation or homogeneous otherwise.

**PARTIAL DIFFERENTIAL EQUATION EXAMPLES**

Some of the examples which follow second-order PDE is given as

1. $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$
2. $u_{xx} + u_{yy} = 0$
3. $ux\frac{\partial^2 u}{\partial x^2} + u^2\, xy\frac{\partial^2 u}{\partial x \partial y} + uy\frac{\partial^2 u}{\partial y^2} + (\frac{\partial u}{(\partial x)})^2 + (\frac{\partial u}{\partial y})^2 + u^3 = 0$

4. $\frac{\partial^2 u}{\partial x^2} + \left(\frac{\partial^2 u}{\partial x \partial y}\right) + \frac{\partial^2 u}{\partial y^2} = x^2 + y^2$

## APPLICATION TO DECOMPOSITION OF ORGANIC MATTER IN 3D SOIL STRUCTURE

## INTRODUCTION

Soil organic matter decomposition is a complex ecological process controlled by the nature of organic matter, the dynamic of microorganisms and the environmental conditions, all in inter action with a heterogeneous structured media. The majority of organic matter models make the assumption that carbon limitation is controlled only by the intrinsic degradability of organic matter. Few of them take also into account the rule of exoenzymes produced by microorganisms to convert complex substrates into available compounds. However, most of organic matter models do not consider the physical heterogeneity that controlled partly the availability of organic matter microorganisms. Recent models simulate the diffusion in soil of soluble carbon substrate in 2D or ID. Some attend were also made to simulate enzyme diffusion in artificial structured environments. However none of these models consider explicitly the real soil structure. The new model of simulates the decomposition of organic matter at the microscale using computer tomography images of real soil but it does not take into account the diffusion process of exoenzymes and dissolved organic matter.

Models available in literature that simulate the biological activity in porous media use different techniques like cellular automata to simulate biomass growth in biofilms, Partial Differential Equations (PDE) to simulate biomass recycling in fungal colonies , multi-agent system to simulate the impact of earthworms on soil structure . In this work we focus on the PDE method because the recent increase performances of computer PDE solvers makes now possible to simulate more and more complex systems. This paper deals with the application of PDE to simulate organic matter degradation in real micro 3D structures taking into account also exoenzymes production and diffusion. We tested our method using high resolution 3D Computed Tomography (CT) of real soil images. Our work faces the problem of solving a nonlinear PDE system (reaction-diffusion) in a non 3D regular mesh.

In the first sections, the paper presents the model of soil organic matter decay using PDE system. The PDE system is formed by reaction diffusion equations. The approximation of the model weak solutions is found by using finite element method and a Newton algorithm. We implement the resolution algorithm by using Freefem3d software. In the last section, the sensitivity of the model according to the dissolved organic matter diffusion and the impact of biological elements distributions in pore space are analyzed using 3D Computed Tomography(CT) image of a real soil sample.

## STATEMENT OF THE MODELING PROBLEM

Our aim is to simulate biological activity in a non regular 3D geometric space.

As specific case study, we deal with microbial decomposition of organic matter in soil. For instance, in this case pore space is described by a set of voxels (cubes) obtained by thresholding a soil sample 3D CT image. Therefore, we take as input data:

- 3D Computed Tomography (CT) image of soil sample providing a set of voxels forming pore space
- Parameters describing the initial spatial distribution of elements involved in biological activity
  - -micro-organisms (MB)
  - -dissolved organic matter (DOM)
  - -fresh organic matter (FOM)
  - -soil organic matter(SOM)
  - -enzymes (ENZ)
  - -inorganic organic matter (CO2)

Thus, we assume classically that the microbial decomposition process involves six biological elements noted MB, FOM, SOM, DOM, ENZ and CO2. We denote MB the micro-organisms secreting enzymes (ENZ). Enzymes decompose organic matters by diffusing through water paths in the pore space. FOM (Fresh Organic Matter) is a kind of organic matter whose decomposition by enzymes is fast, while SOM (Soil Organic Matter) decomposition is slower. DOM (Dissolved Organic Matter) comes from SOM and FOM decomposition. It diffuses through water path to be consumed by MB for their maintenance and growth. It is supposed that MB do not move, so we assume that their diffusion coefficient is very small. Deteriorated enzymes and dead MB are transformed into SOM and DOM. MB breath by producing inorganic carbon ($C0_2$). The output of the simulation system is for each step time the precise distribution of biological activity parameters i.e.:

- MB density
- DOM density
- FOM density
- SOM density
- ENZ density
- $CO_2$ density

Thus, we provide a kind of animated film showing spatially the evolution of biological dynamics characteristics. From these information, we can of course easily compute the classical global evolution curves: $CO_2$ content, DOM content, MB content.

## MODELING BIOLOGICAL DYNAMICS USING PDE: APPLICATION TO SOIL MICROBIAL DECOMPOSITION

### MODEL MATHEMATICAL VARIABLES

Let $\Omega \subset R^3$ be the domain representing the soil pore space. Let $t \geq 0$ be a given time and $x = (x_1, x_2, x_3)^t \epsilon\ \Omega$ be a point of the pore space. Mathematical variables used to model the biological process are :

- $b(x,t)$: $density\ of\ micro-organisms\ (MB)$,
- $n(x,t)$: $density\ of\ DOM$,
- $m_1(x,t)$: $density\ of\ SOM$,
- $m_2(x,t)$: $density\ of\ FOM$,
- $e(x,t)$: $density\ of\ enxymes\ and$
- $c(x,t)$: $density\ of\ co2$

We point out that x denote a vector of $R^3$ representing the coordinates of a point in the 3D affine space.

## PARTIAL DIFFERENTIAL EQUATIONS RULING MODEL VARIABLES

### MICRO-ORGANISMS (MB)

Let V be an elementary volume in $\Omega$ . The variation of the quantity of microbial decomposers (MB) in v is due to :

- Micro-organisms diffusion,
- Micro-organisms growth,
- Micro-organisms mortality,
- Micro-organisms breathing,
- Enzymes production.

Thus during the breathing and the enzymes production , microbial decomposers lose a part of their masses. The following equation summarizes the process.

$$variation\ of\ b = diffusion\ of\ b + growth\ of\ b - mortality, breathing\ enzymes$$

We assume that the microbial decomposers (MB) growth depends on the quantity of dissolved organic matter. Indeed, the micro-organisms (MB) consumed the dissolved organic matter which is provide by the decomposition of organic matter (FOM,SOM.....) by enzymes , we use the Monod equation:

$$\frac{\partial b}{\partial t} = \frac{Kn}{K_b + n} b\ ,$$

where the variables are set as follows:

$b$: $concentration\ of\ microbial\ decomposers$

$K$: $maximal\ growth\ rate$

$k_b$: $half\ saturation\ constant$

$n$: $concentration\ of\ dissolved\ organic\ matter.$

The above equation describes microbial decomposers growth rate. Thus, the variation of b is expressed by the following equation:

$$\frac{\partial b}{\partial t} = D_b \Delta_b + \left(\frac{kn}{k_b + n} - \mu - r - v\right) b$$

where $D_b$ represents the microbial decomposers diffusion coefficient, $\mu$ is to the mortality rate , r is set to the breathing rate and $v$ represents the enzymes production rate.

**DISSOLVED ORGANIC MATTER: (DOM)**

DOM density variation comes from:

- DOM diffusion
- Consumption of DOM by microbial decomposers
- SOM and FOM transformation by enzymes into DOM
- Microbial decomposers mortality (dead micro-organisms become partly DOM and partly SOM)
- Enzymes degradation (enzymes become partly DOM and partly SOM)

Thus , the following equation summarizes DOM density variation process:

Variation of n = diffusion of n + transformation of $m_1, m_2 + mortality\ of\ b$ $-growth\ of\ b$(consumption ) + degradation of e

We assume that SOM and FOM transformation rate into DOM is a growing function of enzymes density. So we use the following equation to express organic matter transformation rate:

$$\frac{\partial m}{\partial t} = \frac{ce}{k_m + e} m$$

Where:

C is the maximal transformation rate

e represents enzymes concentration

Organic matter concentration is noted m

$k_m$ represents the half-saturation constant.

DOM variation is ruled by the following equation:

$$\frac{\partial n}{(\partial t)} = D_n \Delta_n + \frac{e}{k_m + e}(c_1 m_1 + c_2 m_2) - \frac{kn}{k_b + n} b + \alpha_1 e + \alpha_2(\mu) b,$$

where $D_n$ represents DOM diffusion coefficient. $c_1$ and $c_2$ represent respectively SOM and FOM maximal transformation rate. $\alpha_1$(C) represents the transformation rate of deteriorated enzymes into DOM. $\alpha_2(\mu)$ is the transformation rate of dead microbial decomposers (MB) into DOM.

**SOIL ORGANIC MATTER: (SOM)**

SOM quantity variation comes from the transformation of a part of SOM into DOM, by enzymes degradation and by MB mortality.

$$variation\ of\ m_1 = -transformation\ of\ m_1 + degradation\ of\ e + mortality\ of\ b$$

Thus, SOM quantity variation is expressed by the following equation:

$$\frac{\partial m_1}{\partial t} = -\frac{c_1 e}{k_m + e} m_1 + \left(1 - \alpha_1(c)\right) e + \left(1 - \alpha_2(\mu)\right) b.$$

where 1-$\alpha_1$ (C) is the rate of deteriorated ENZ transformed into SOM and 1-$\alpha_2(\mu)$ is the rate of dead MB transformed into SOM.

**FRESH ORGANIC MATTER: (FOM)**

FOM quantity variation is caused by its transformation by enzymes into DOM as follows:

$$variation\ of\ m_2 = -transformation\ of\ m_2$$

FOM quantity variation is expressed by the following equation:

$$\frac{\partial m_2}{\partial t} = -\frac{c_2 e}{k_m + e} m_2$$

**ENZYMES (ENZ)**

Enzymes quantity variation is due to the enzymes production by microbial decomposers (MB), the enzymes diffusion, and the enzymes degradation:

$$variation\ of\ e = diffusion\ of\ e + production\ of\ e - degradation\ of\ e$$

The above scheme is expressed by the following equation:

$$\frac{\partial e}{\partial t} = D_e \Delta_e + vb - \mathrm{C}e$$

where $D_e$, is the enzymes diffusion coefficient, $\mathrm{C}$ represents the enzymes degradation rate.

**INORGANIC CARBON : ($CO_2$)**

Inorganic carbon ($CO_2$) variation is due to its diffusion and to its production by microbial decomposers (MB) during breathing.

$$variation\ of\ c = diffusion\ of\ c + production\ of\ c.$$

The CO2 evolution equation is :

$$\frac{\partial c}{\partial t} = D_c \Delta_c + rb_1$$

Where $D_c$ is set to the $CO_2$ diffusion coefficient.

**BOUNDRY CONDITIONS FOR MODEL VARIABLES**

On the border of $\Omega$ noted $\partial\Omega$ , we use the Neumann boundary conditions. It means that flow is null on $\partial\Omega$ for all variables.

**FORMING PARTIAL DIFFERENTIAL EQUATIONS (PDE) SYSTEM**

In this section we use the above equations describing variables variations to set a global PDE system modeling biological dynamics. Let T> 0 be a fixed time and let's define

$$\Omega_T = \Omega \times (0, T).$$

Therefore, the whole system of partial differential equations governing the biological model becomes in $\Omega_T$ :

$$\frac{\partial b}{\partial t} = D_b \Delta_b + \left(\frac{kn}{k_b + n} - \mu - r - v\right) b,$$

$$\frac{\partial n}{\partial t} = D_n \Delta_n + \frac{e}{k_m + e}(c_1 m_1 + c_2 m_2) - \frac{kn}{k_b + n} b + \alpha_1(\mathrm{C})e + \alpha_2(\mu) b,$$

$$\frac{\partial_{m1}}{\partial t} = -\frac{c_1 e}{k_m + e} m_1 + (1 - \alpha_1(\mathrm{C})e + \left(1 - \alpha_2(\mu)\right) b,$$

$$\frac{\partial m_2}{\partial t} = -\frac{c_2 e}{k_m + e} m_{2,}$$

$$\frac{\partial e}{\partial t} = D_e \Delta_e + vb - \mathrm{C}e,$$

$$\frac{\partial c}{\partial t} = D_c \Delta_c + rb$$

We use neumann homogeneous boundary conditions and the following initial conditions in $\Omega$:

- $b_0(x)$ for MB,
- $n_0(x)$ for DOM.
- $m_{10}(x)$ for SOM,
- $m_{20}(x)$ for FOM,
- $e_0(x)$ for ENZ,
- $c_0(x)$ for CO2.

Therefore, the above PDE system describes precisely microbial decomposition of organic matter in soil. In the following section, we show how to solve this PDE system which allows to practically simulate soil biological activity.

**NUMERICAL RESOLUTION OF THE PDE SYSTEM (model):** Soil biological dynamics simulation

**PDE VECTORIAL SYSTEM FORMULATION**

We simplify the system writing by transforming it into a vector form. Let's define vectors the following way:

$$u = (u_1, u_2, u_3, u_4, u_5, u_6)^t$$

$$= (b, n, m_1, m_2, e, c)^t$$

$$u_0 = (b_0 n_0 m_{10} m_{20} e_0 c_0)^T$$

The diffusion coefficients matrix D is defined as follows:

The reaction terms of equations are represented by functions $F_i, i = 1,2, \ldots ,6$ defined as follows:

- $F_1(u) = \left(\frac{Ku_2}{K_s+u_2} - \mu - r - v\right) u_{1,}$
- $F_2(u) = \frac{Ku_5}{K_m+u_5}(c_1 m_1 + c_2 m_2) - \frac{Ku_2}{k_s+u_2} u_1 + \alpha_1(\mathrm{C}) u_5 + \alpha_2(\mu) u_1,$
- $F_3(u) = \frac{c_1 u_5}{K_m+u_5} u_3 + \left(1 - \alpha_1(\mathrm{C})\right) u_5 + \left(1 - \alpha_2(\mathrm{C})\right) u_1,$
- $F_4(u) = -\frac{c_1 u_5}{K_m+u_5} u_4,$
- $F_5(u) = v u_1 - \mathrm{C} u_5,$
- $F_6(u) = r u_1.$

Let's define the vector function F such that

$$F(u) = (F_1(u), F_2(u), F_3(u), F_4(u), F_5(u), F_6(u))^T$$

The vector form of the system is

$$\{\partial_t u = div(D \nabla u) + F(u) \qquad in\ \Omega_T,$$

$$\frac{\partial u}{\partial t} = 0 \quad on\ \partial\Omega \times ]0, T[,$$

$$u(t = 0) = u_0\ \ in\, \Omega.$$

**SYSTEM VARIATIONAL FORMULATION**

Let's introduce the following Sobolev space

$$V = \left\{ v \in \left(H^1(\Omega)\right) : \frac{\partial u}{\partial n} = 0\ \ sur\ \partial\Omega \right\}.$$

Assuming that the data are sufficiently regular , the variational formulation consist in finding a function u(t)$\in V\ such\ that$ :

$$\int_\Omega \frac{\partial u}{\partial t} v dx + \int_\Omega D \nabla_u \nabla_v dx = \int_\Omega F(u) v dx \qquad \forall v \in V$$

After building a mesh $\Omega_h$ of domain $\Omega$ we solve variational formulation consists in the following discrete space:

$$V_h = \left\{ v \in \left(C(\Omega)\right)^6 :\ \forall K \in \Omega_h\ \ (v|k \in P_1)^6 \right\}$$

**RESOLUTION SCHEME OF THE VARIATIONAL SYSTEM**

**NUMERICAL SCHEME**

The numerical resolution of the problem is divided into three steps:

STEP 1: we discretize the problem via finite element method in the finite dimensional space $V_k$. The problem consists in solving the following system

$$\frac{\partial U}{\partial t} + BU = F(U).$$

Where

$$B_{i,j} = \int_\Omega D \nabla_{\emptyset_i} \nabla_{\emptyset_j} dx \quad \forall i, j = 1,2, \ldots, Ndof.$$

And

$$F(U)_i = \int_\Omega F_i\,(U) \emptyset_i dx \qquad \forall i = 1,2, \ldots\ldots, Ndof.$$

N dof is the number of freedom degrees.

The sequence $(\emptyset_i) 1 \leq i \leq N\ dof\ defines$ the basis of the space $V_h$.

STEP 2: we use implicit scheme to discritize time.

Let $N_t\ be$ a positive integer denoting the number of time steps.

We call the step time $\delta t = \frac{T}{N_t}$.

The numerical scheme is :

$$\frac{U^n - U^{n-1}}{\delta t} + BU^n = F(U^n), \qquad n \in N^*$$

Which can be as follows :

$$(I + \delta t\, B)U^n = U^{n-1} + \delta t F(U^n), \qquad n \in N^*$$

Where:

I is the identity matrix

$U^n$ is the solution at time n*$\delta t$.

STEP 3 : We linearize the function F($U^n$). We ignore $U^n$ at the time n*$\delta t$ . Therefore , we note $U^*$ the approximation of $U^n$ and $\delta U$ the correction term. We obtain :

$$U^n = U^* + \delta U,$$

So the numerical scheme becomes:

$$(I + \delta t\, B)\delta U = U^{n-1} - U^* + \delta t(F(U^n) - BU^*).$$

We consider the following approximation :

$$F(U^n) \cong F(U^*).$$

The numerical scheme becomes:

$$(I + \delta t\, B)\delta U = U^{n-1} - U^* + \delta t(F(U^*) - BU^*).$$

**RESOLUTION ALGORITHM**:

From the initial conditions, we know $U^0$. Then in order to find solution $U^n$ knowing $U^{n-1}$,we initialize $U^*$ *to* $U^{n-1}$. Afterwards, we repeat the numerical scheme by replacing the term $U^*$by $U^* \leftarrow U^* + \delta U$ after each iteration. Two stopping criteria are defined:

- We define a maximal number of iterations called N max
- We define a very small positive real $\in_a$and we stop the loop if the norm of $\delta$U is smaller than $\epsilon_a$

When one of the stopping criteria is satisfied, $U^*$ represents the searched solution. Thus, the resolution algorithm can be described as follows:

1. N max and $\epsilon_a$ are fixed.
2. $n \geq 1, U^{n-1}$ is known.
   (a) $U^* \leftarrow U^{n-1}$,
   (b) The beginning of the loop for linearization
1. Solve $(1 + \delta t B)\delta U = U^{n-1} - U^* + \delta t(F(U^*) - BU^*)$,
2. Update $U^* \leftarrow U^* + \delta U$,
3. If the condition on $\epsilon_a$ or Nmaz is satisfied, the loop is stopped. (c) $U^n \leftarrow U^*$

We give in annex 7 the Freefem3d code we implement for solving the system. In many practical cases, pore space is described by a too high number of voxels to provide reasonable computing and memory costs to solve the system. To tackle this problem, we use an octree data structure for describing pore space. We give in annex 8 the practical implementation in code C++.

## EXPERIMENTAL RESULTS USING 3D CT SOIL IMAGES : NUMERICAL SIMULATIONS

### SOIL SAMPLE AND PARAMETERS

The soil columns were scanned by means of a high resolution micro-CT machine (SIMCT Equipment: SIMBIOS, University of Abertay Dundee, Scotland) operating at 90KeV and a current of 112mA. The soil is sampled in the surface layer (0-20cm) of an agricultural field at the INRA experimental site of Feucherolles (FEU) (50km west of Paris) in France. It is a loamy soil typic hapludalf (15% clay, 78% loam and 7% sand). We use 3D Computed Tomography (CT) image of a soil sample having the following resolution: 63$\mu m$ x 63m$\mu$ x 63$\mu$m. The size of the image used for the simulations is 256x256x256 voxels that corresponds to 16128$\mu$mx 16128$\mu$m x 16128$\mu$m. The size of our sample is about $4cm^3$ . We obtain pore space by thresholding CT image using When a voxel gray level is less than a given threshold, it is assumed to belong to pore space. The pore space forms 34% of the soil sample and is totally filled with water. In order to reduce the number of voxels, we apply the octree method up to the level 3 of the octree structure.

Same as in real conditions, we initially introduce 2.5gC/kg of dry soil for DOM in a soil containing 250mgC/kg soil of microorganisms. In our sample of 4cm, we introduce 10000µg of carbon-DOM for 1000µg of carbon-MB initially contained in the soil. For the following experiments, we assume that at the initial time, there is only microorganisms MB and dissolved organic matter DOM. MB do not produce enzymes ENZ (n equals 0) and dead MB are transformed into SOM but not into DOM ($\alpha_2$ equals 0).

The maximal growth rate of microorganisms k, the mortality rate m and the breathing rate r were set respectively to the values of $0.7\text{day}^{-1}, 0.001\text{day}^{-1}$ and $0.02\text{day}^{-1}$ taken in Monga et al (2008). The half saturation constant $K_1$ were taken in Ingwersen et al., (2008) with the value of 0.264mgCg-$^{1}$.

In the following simulations, the soil is saturated of water. In real conditions the saturation may lead to anaerobic process. Indeed, Hojberg et al. (1994) showed that oxygen can diffuse only until millimeters inside saturated aggregates. If we assume that our sample, whose larger dimension was 16mm, was surrounded by air-filled pores, the oxygen may penetrate inside. by the saturated pores. We found also large pores with diameter higher than 500µm in the sample (see figure 4). In our simulations, we assume aerobic conditions for decomposition. In future version of the model, we will add oxygen consumption.

## CONCLUSION

In this paper we have discussed about partial differential equation and classifications, types of partial differential equations and also, in this study we proposed a new model of organic matter decomposition in the soil pores. The novelties of our approach are to consider the real 3D microstructures of soil, to take into account for enzyme production/diffusion and to simulate accessibility of dissolved organic matter to microorganisms. We applied our model to real CT images of soil in which we add realistic amount of organic matter and microorganisms. Sensitivity analysis of the model to distance between microorganisms and organic matter shows the impact of pore connectivity in the decomposition process that would not easily to visible without our 3D approach. In future studies, simulations with real experimental data will be carried out with our model in order to analyse microbial competition of degradation under different water contents.

## BIBLIOGRAPHY

1. Basic concepts – FAMU-FSU college of engineering
2. Allison steven d. Cheaters, diffusion and nutrients constrain decomposition by microbial enzymes in spatially structured environments . Ecology letters,8(6):626-635, june 2005.
3. 3. del Pino S. And Pironneau O.Asymptotic analysis and layer decomposition for the complex exercise Computational Geosciences, 8:149-162,2007.
4. Dobrzynski C. and Frey P.J. and Mohamadi B.and Pironneau O. Fast and accurate simulation of air –cooled structures. Computer methods applied in Mechanical Engineering . 195:3168-3180,2006.
5. Falconer RE, Bown JL, White NA, Crawford JW . Biomass recycling :a key to efficient firaging by fungal colonies . OIKOS., 116 (9):1558-1568,2007.
6. Fang C , Smith P , Smith JU, Moncrieff JB. Incorporating microorganisms as decomposers into models to simulate soil organic matter decomposition . Geoderma .,129 (3-4) : 139-146, 2005
7. Ingwersen Joachim, Poll Christian, Streck Thilo, and Kandeler Ellen. Micro-scale modelling of carbon turnover driven by microbial succession at a biogeochemical interface. Soil biology and biochemistry, 40(4):864-878, 2008.
8. Long T. and Or D. Aquatic habitats and diffusion constraints affecting microbial coexistence in unsatured porous media. Water Resources Research, 41, August 2005.
9. Marilleau N, Cambier C, Drogoul A, Chotte JL, Perrier E, Blanchart E. Multiscale MAS modelling to simulate the soil environment: Application to soil ecology. Simulation modelling practice and theory., 16(7): 736-745, 2008.